



# Digital Holographic Microscopy

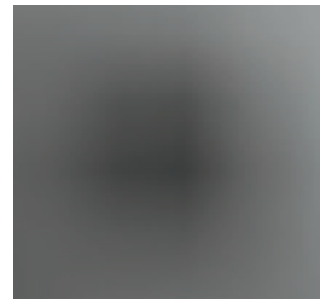
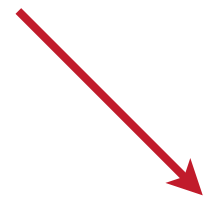
Irwin Zaid

Junior Research Fellow, Christ Church & Physics

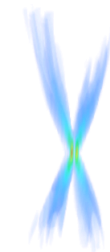
# Digital Holographic Microscopy



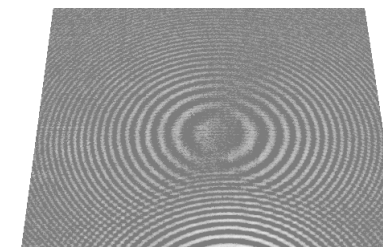
The present...



Bright-field image of a gold nanoparticle



The world of tomorrow!



Hologram of, and 3D electromagnetic field around, a gold nanoparticle

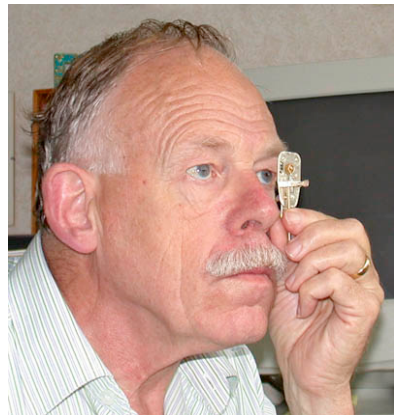
# The Problem with Microscopy



- Most microscopes are limited to **2D** snapshots of the **intensity** in a single focal plane



van Leeuwenhoek

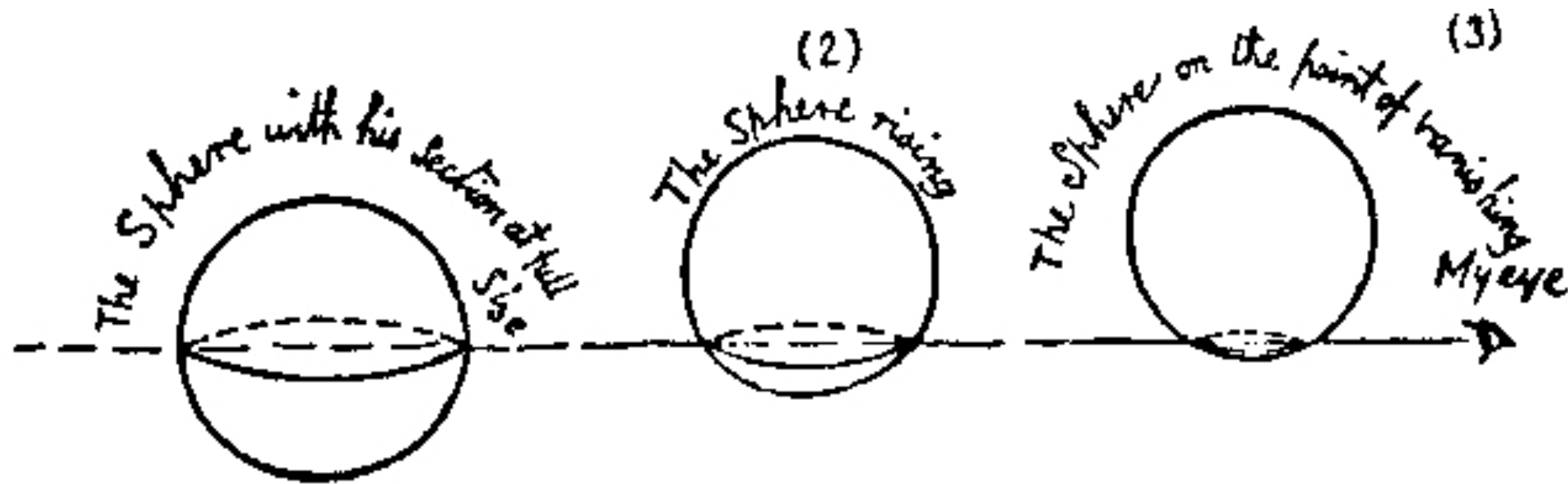


Microscopy in the 17th century



Microscopy today

# Beyond Flatland





# Holography



- Holography is able to reconstruct in 3D the **electromagnetic field** scattered off an object



Gabor

# Not Holograms



# What Holography Actually Is

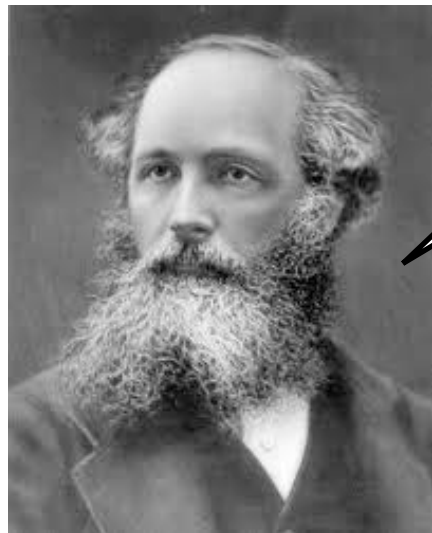
- Holography is about interference



- To record a hologram, interfere the light scattered by an object with a reference beam
- To reconstruct a hologram, shine a copy of the reference beam on the recording (**classical holography**) or numerically propagate the scattered light backwards (**digital holography**)

# Holography in One Slide

- If we know how light propagates in a medium, then the 3D electromagnetic field can be reconstructed from a single 2D slice



Maxwell

$$\begin{aligned} \nabla \cdot \mathbf{D}(\mathbf{r}, t) &= \rho(\mathbf{r}, t) \\ \nabla \times \mathbf{E}(\mathbf{r}, t) &= -\frac{\partial}{\partial t} \mathbf{B}(\mathbf{r}, t) \\ \nabla \cdot \mathbf{B}(\mathbf{r}, t) &= 0 \\ \nabla \times \mathbf{H}(\mathbf{r}, t) &= \frac{\partial}{\partial t} \mathbf{D}(\mathbf{r}, t) + \mathbf{j}(\mathbf{r}, t) \end{aligned}$$

Monochromatic wave  
Homogeneous and isotropic medium  $\rightarrow \nabla^2 U(\mathbf{r}, t) + k^2 U(\mathbf{r}, t) = 0$

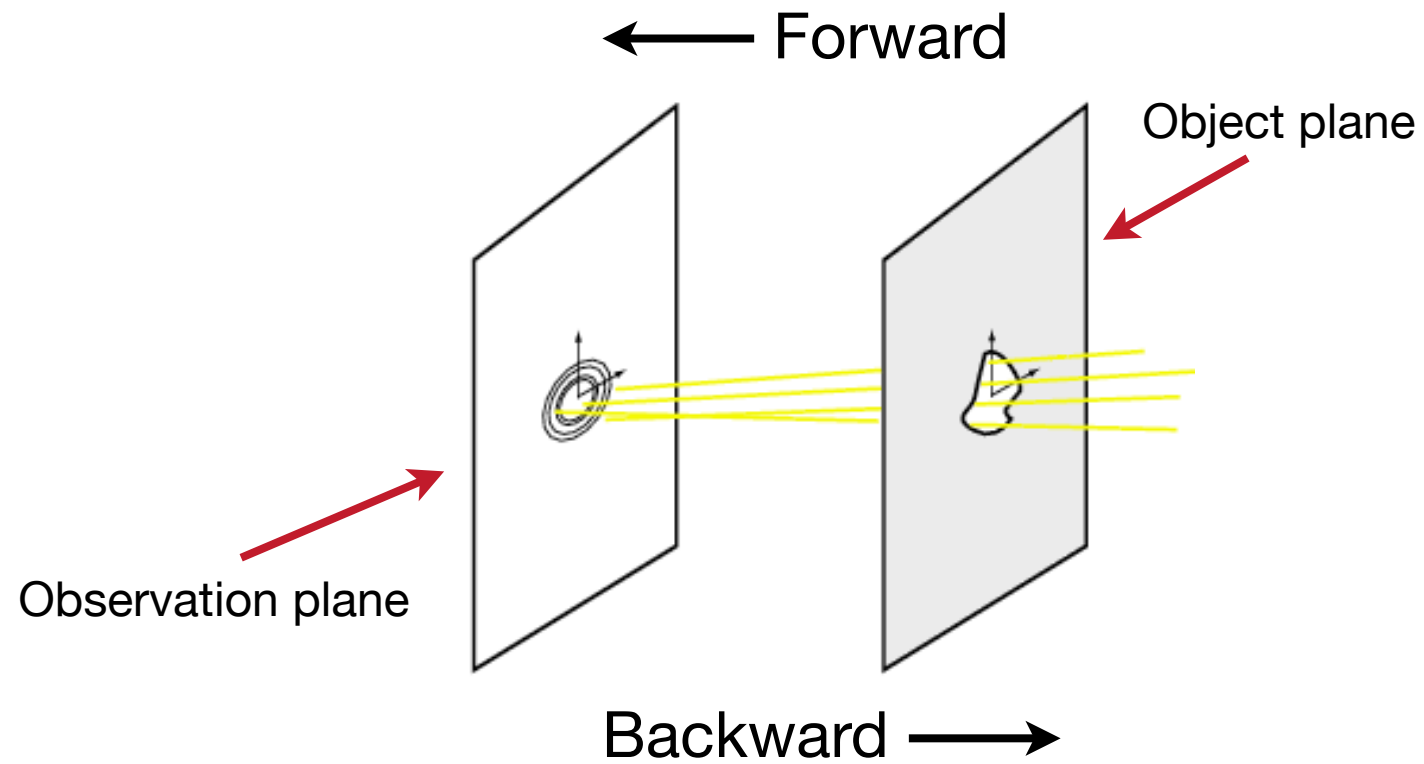
$$U(x, y, z) = \int g(x - x', y - y', z) U(x', y', 0) dx' dy' \quad \text{with} \quad g(x, y, z) = \frac{ze^{ikr}}{r^2} \left( \frac{1}{2\pi r} - \frac{i}{\lambda} \right)$$

# Rayleigh-Sommerfeld Diffraction

$$U(x, y, z) = \int g(x - x', y - y', z) U(x', y', 0) dx' dy' \quad \text{with} \quad g(x, y, z) = \frac{ze^{ikr}}{r^2} \left( \frac{1}{2\pi r} - \frac{i}{\lambda} \right)$$

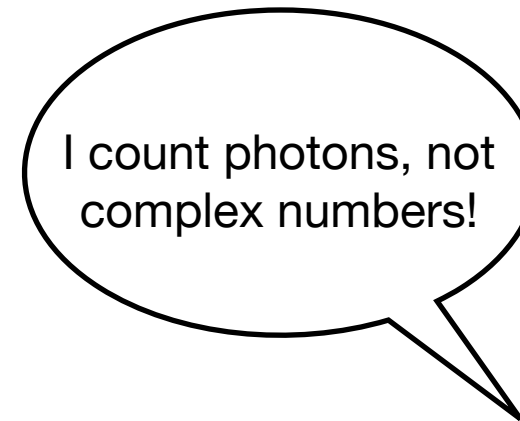
↑
↑
↑

Field at z                      Propagator                      Field at z = 0



# How to Record a Complex Field?

- A hologram encodes the amplitude and phase of an electromagnetic wave



$$I(x, y) = |R(x, y) + O(x, y)|^2 = |R(x, y)|^2 + |O(x, y)|^2 + R(x, y)O(x, y)^* + R(x, y)^*O(x, y)$$

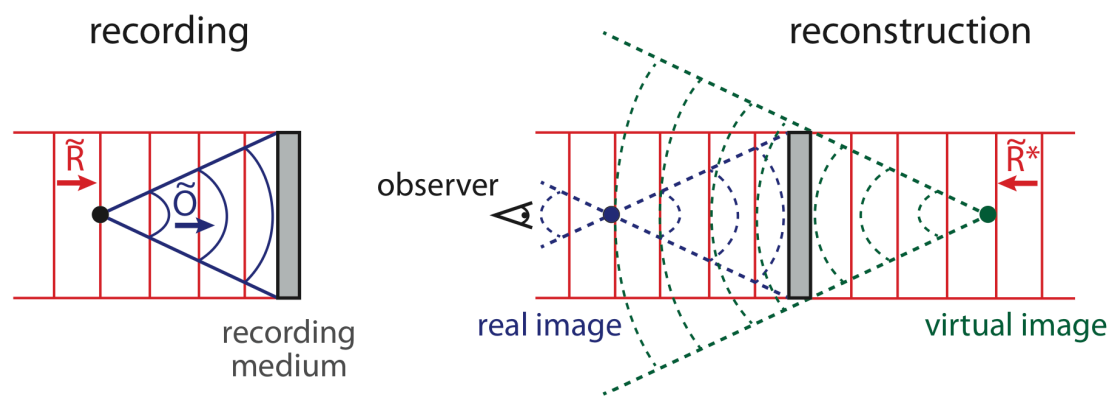
Reference wave (points to  $R(x, y)$ )  
 Reference intensity (points to  $|R(x, y)|^2$ )  
 Hologram (points to  $I(x, y)$ )  
 Object wave (points to  $O(x, y)$ )  
 Object intensity (points to  $|O(x, y)|^2$ )  
 What we really want... (points to  $R(x, y)O(x, y)^* + R(x, y)^*O(x, y)$ )

$$\frac{I(x, y)}{|R(x, y)|^2} = 1 + \left| \frac{O(x, y)}{R(x, y)} \right|^2 + \frac{e^{i\phi(x, y)} O(x, y)^* + e^{-i\phi(x, y)} O(x, y)}{|R(x, y)|} \quad \text{with} \quad R(x, y) = |R(x, y)|e^{i\phi(x, y)}$$

# Flavours of Holography

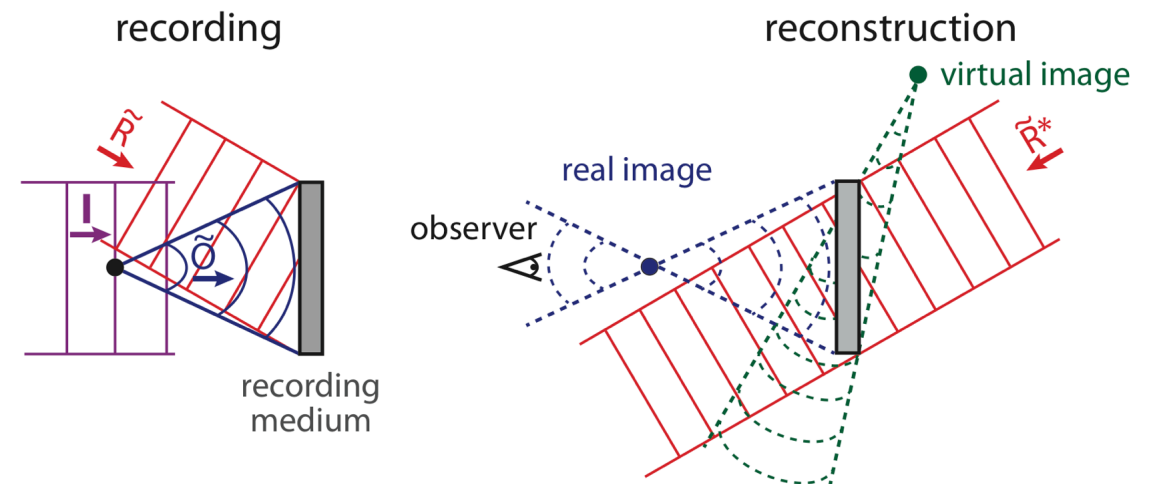


## Inline holography



$$R(x, y) = |R(x, y)|$$

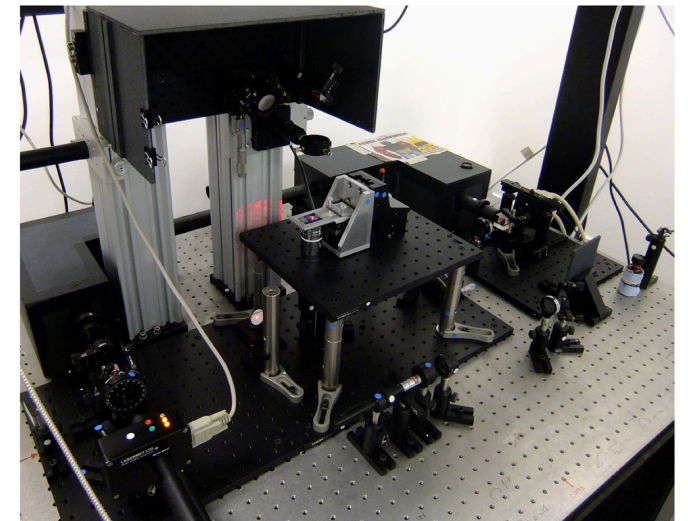
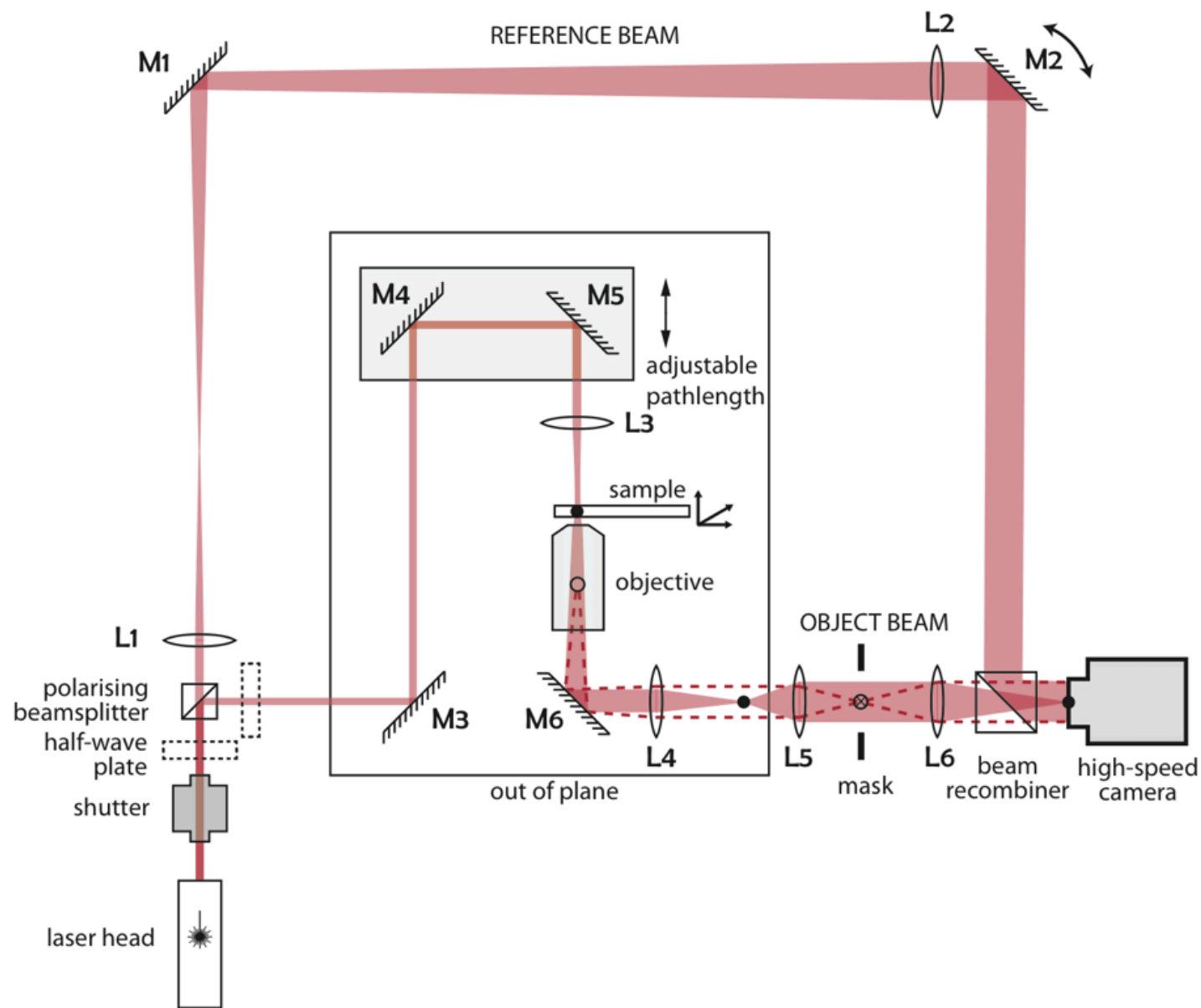
## Off-axis holography



$$R(x, y) = |R(x, y)|e^{-ikx \sin \theta}$$



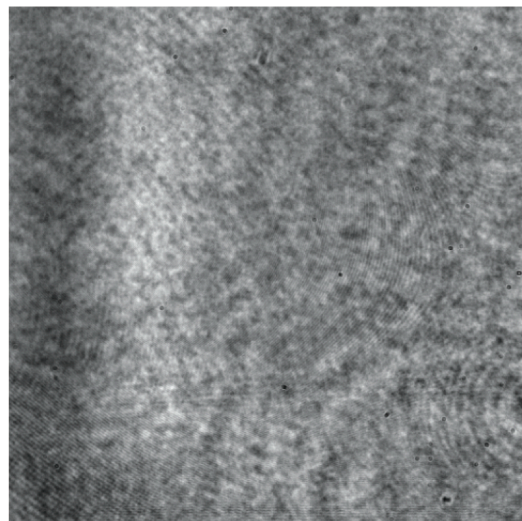
# A Digital Holographic Microscope



- ~60 nm spacing in x and y
- 2000 FPS at 512 px by 512 px

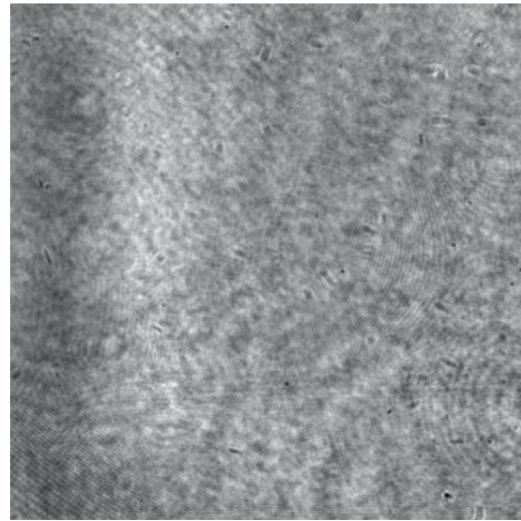


# Inline Recording



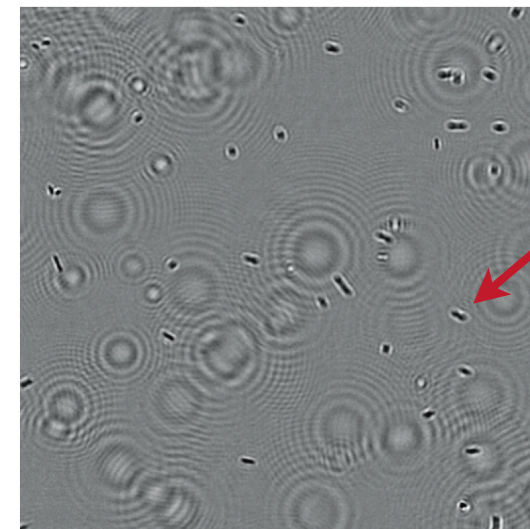
Raw hologram

÷

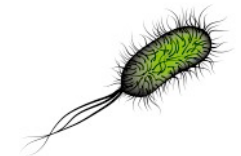


Background hologram

=



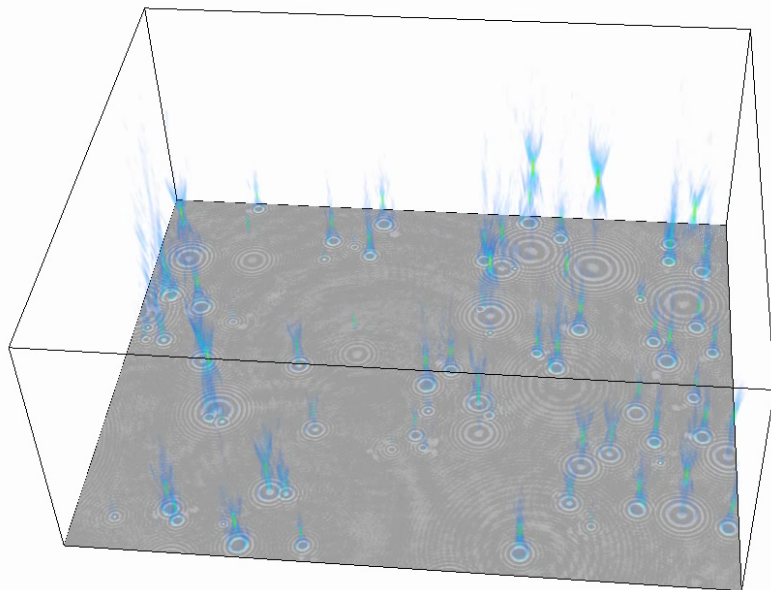
Normalized hologram



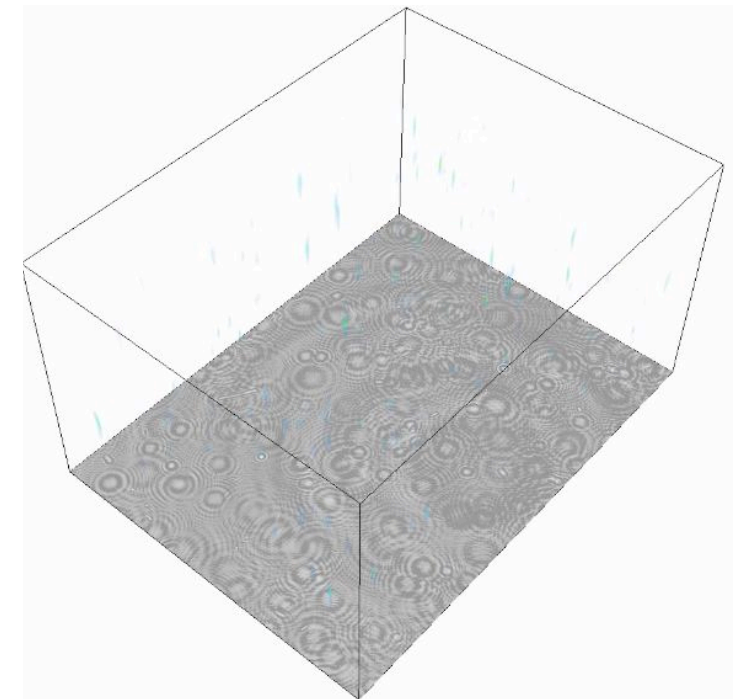
*E. coli*

$$\frac{I(x, y)}{|R(x, y)|^2} = 1 + \left| \frac{O(x, y)}{R(x, y)} \right|^2 + \frac{O(x, y)^* + O(x, y)}{|R(x, y)|} \quad \text{with} \quad R(x, y) = |R(x, y)|$$

# Inline Reconstructions

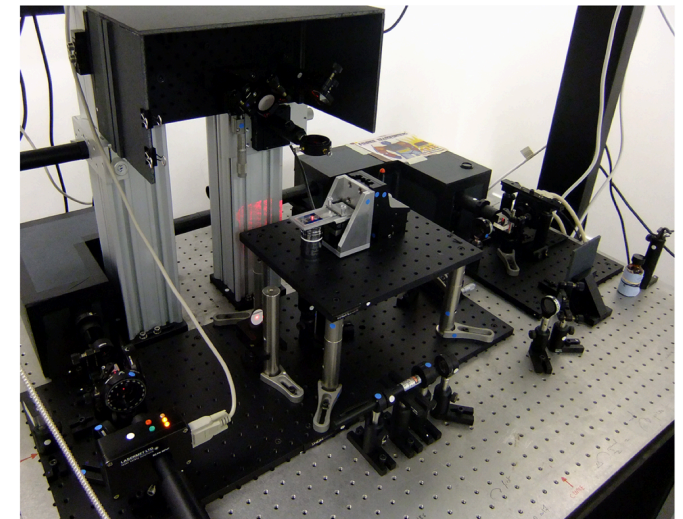
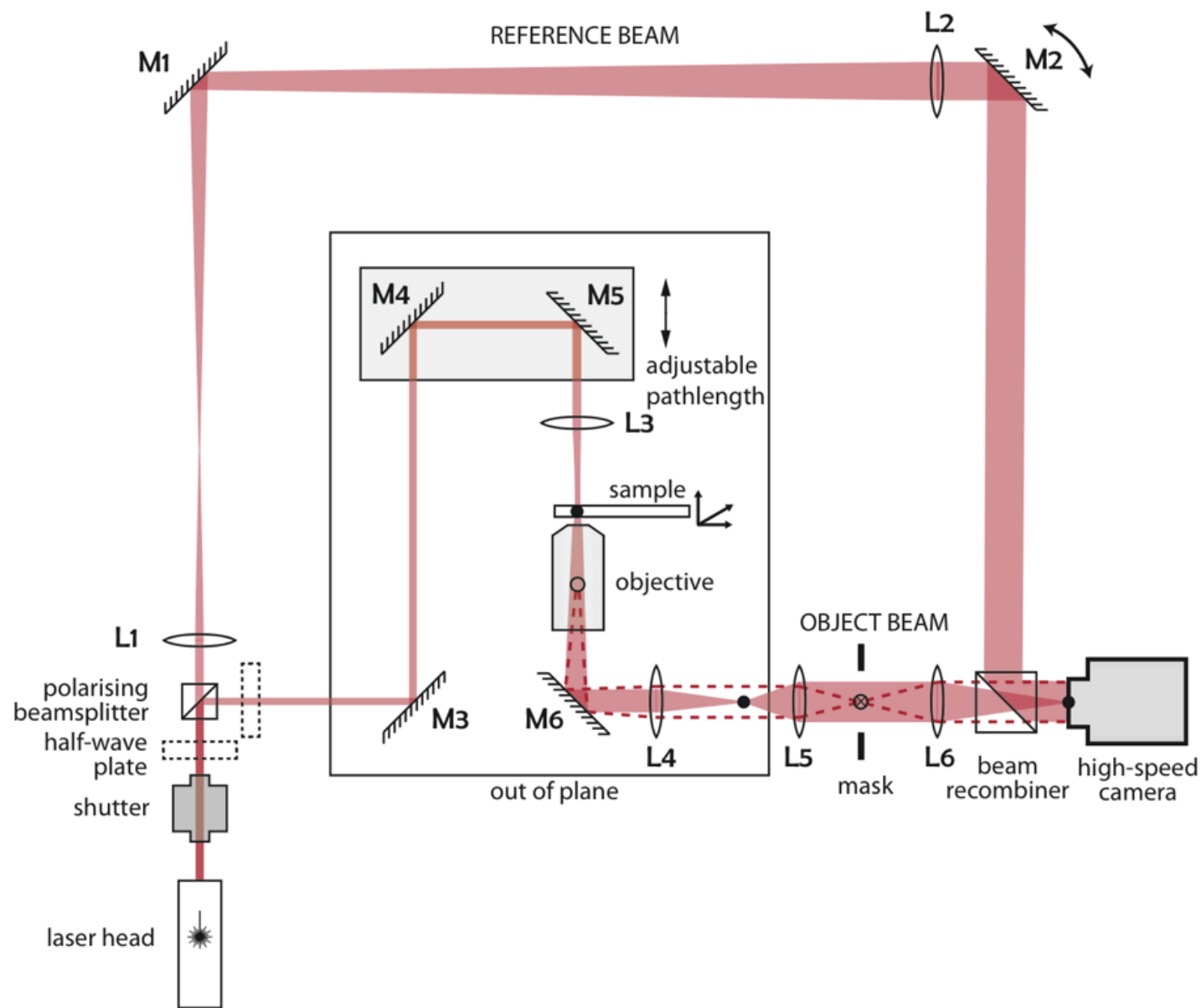


Diffusing  $1\ \mu\text{m}$  particles in a volume of  $180\ \mu\text{m}$  by  $140\ \mu\text{m}$  by  $70\ \mu\text{m}$



Swimming *E. coli* cells in a volume of  $200\ \mu\text{m}$  by  $160\ \mu\text{m}$  by  $100\ \mu\text{m}$

# A Digital Holographic Microscope

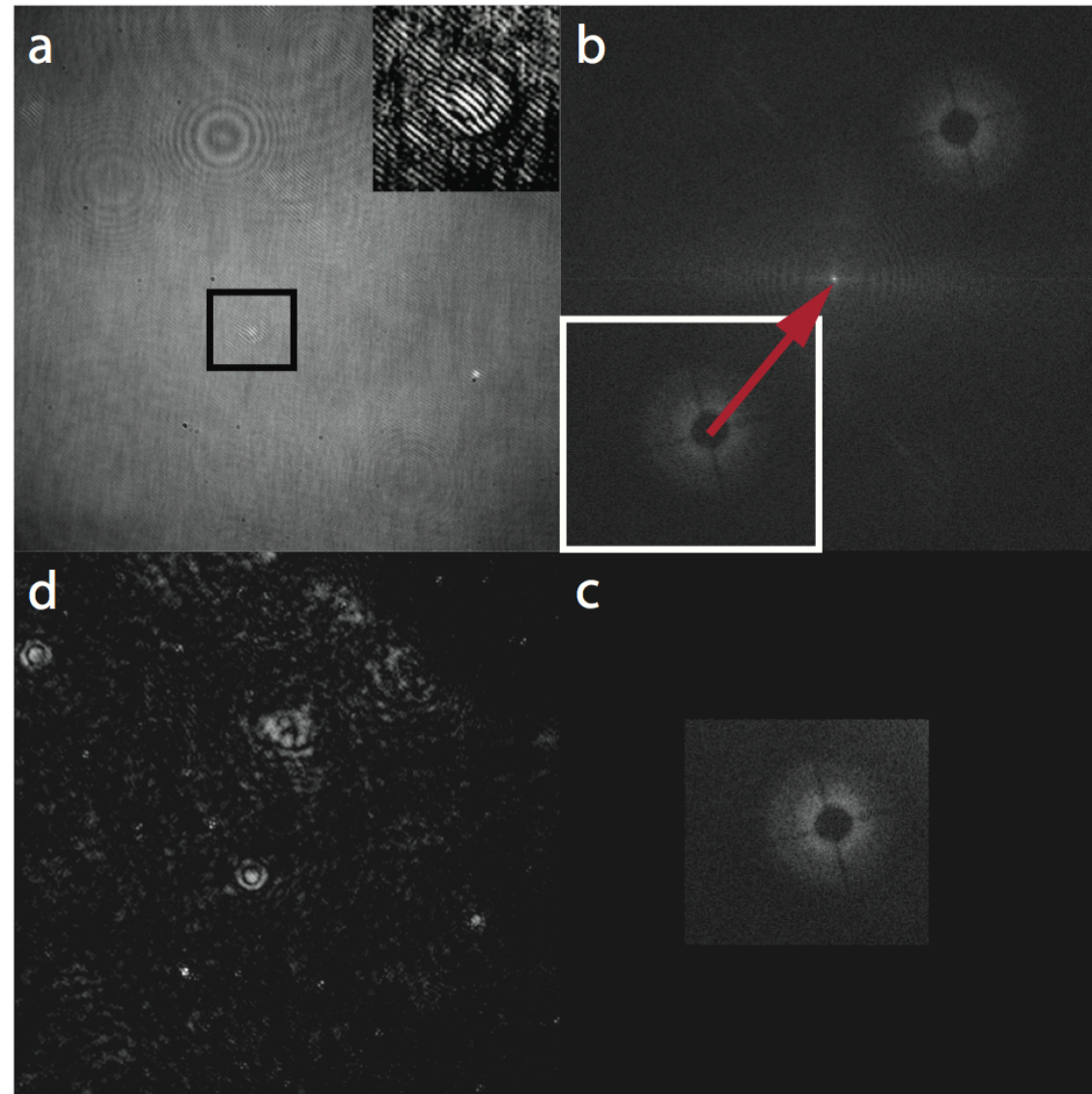


- ~60 nm spacing in x and y
- 2000 FPS at 512 px by 512 px



# Off-axis Recording

Hologram



Fourier transform

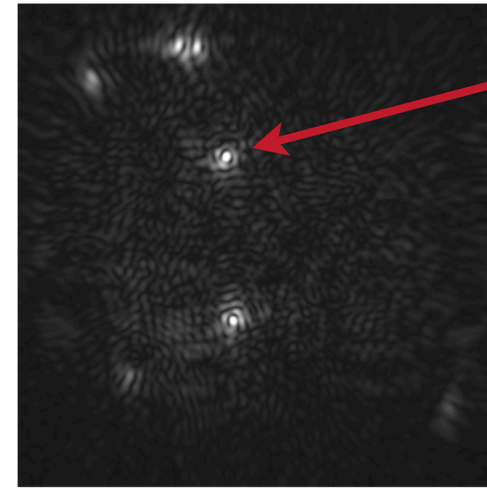
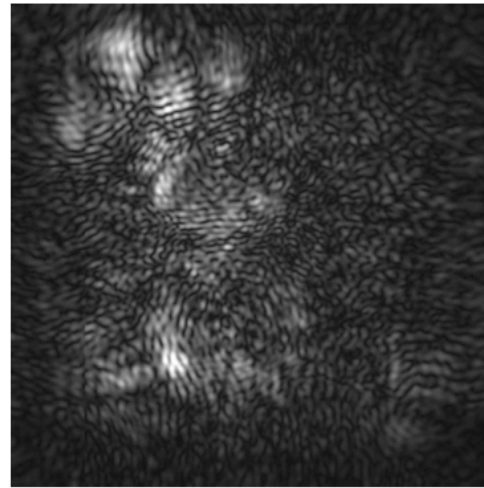
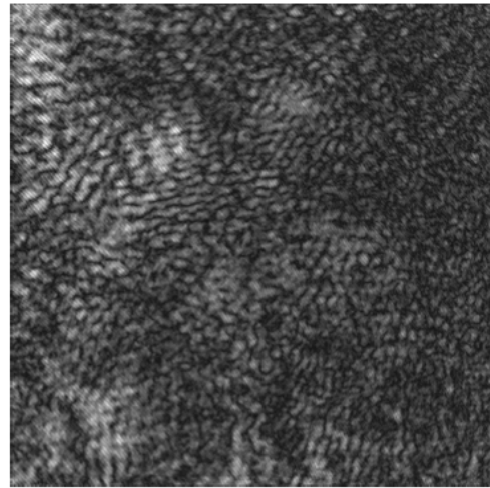
Filtered hologram


Cropped and shifted  
Fourier transform

$$\frac{I(x, y)}{|R(x, y)|^2} = 1 + \left| \frac{O(x, y)}{R(x, y)} \right|^2 + \frac{e^{-ikx \sin \theta} O(x, y)^* + e^{ikx \sin \theta} O(x, y)}{|R(x, y)|} \quad \text{with} \quad R(x, y) = |R(x, y)| e^{-ikx \sin \theta}$$

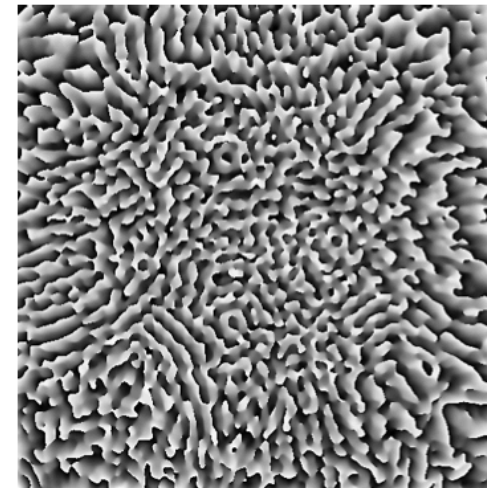
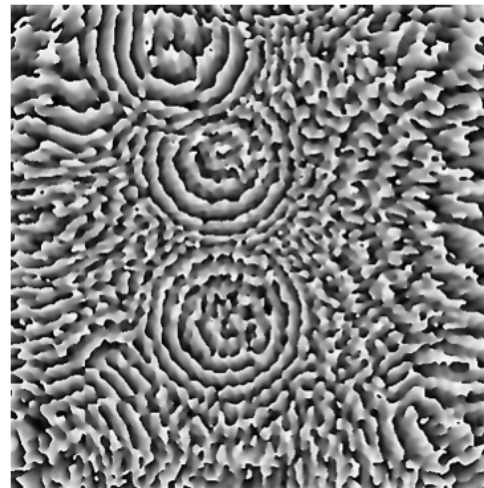
# Off-axis Reconstructions

Amplitude



  
1 μm bead

Phase



$z = 0 \mu\text{m}$

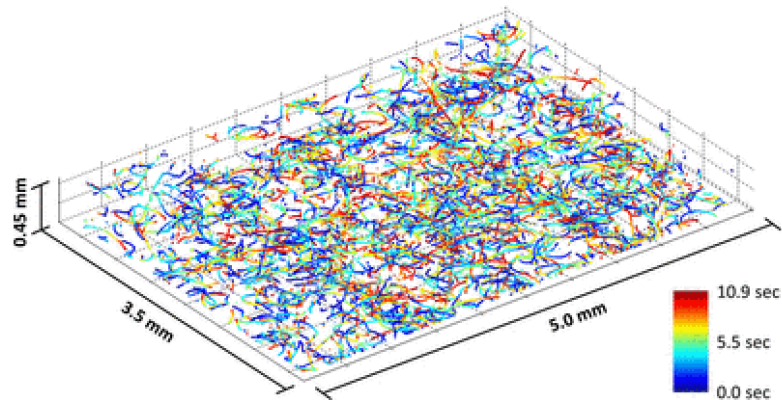
$z = 13 \mu\text{m}$

$z = 21.5 \mu\text{m}$



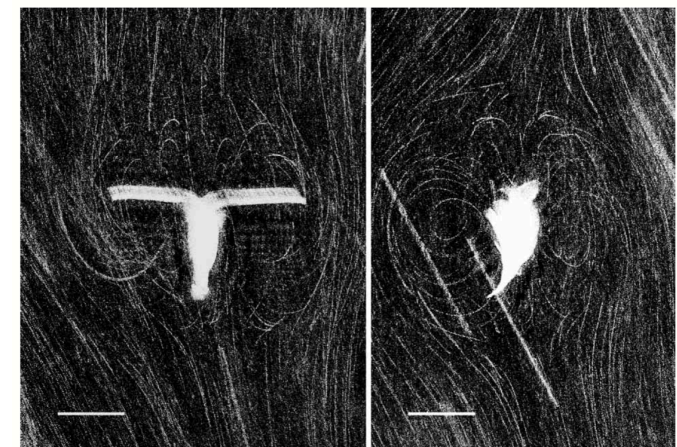
# Why You Should Bother...

Motility of human sperm



T.-W. Su *et al*, PNAS (2013)

Fluid flow around a copepod



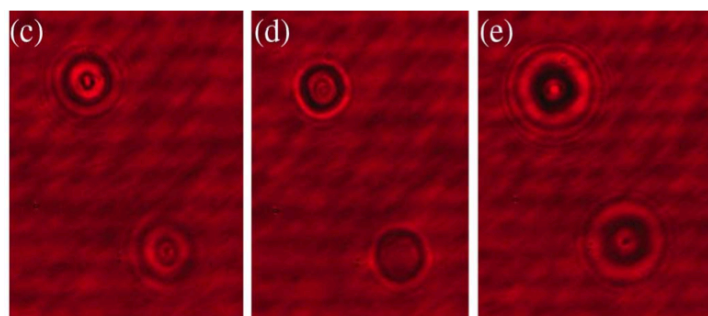
E. Malkiel *et al*, J. Exp. Bio. (2003)

3D imaging of biological samples



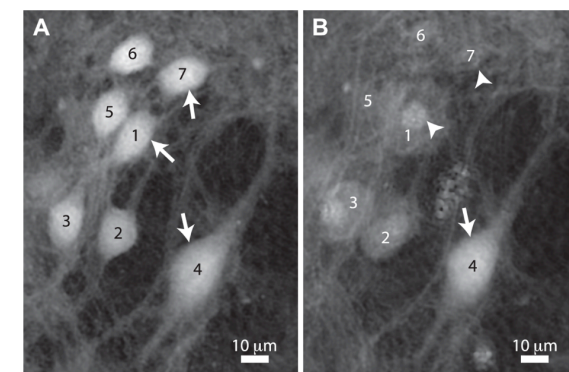
S. Bernet *et al*, Opt. Express (2011)

Differentiation of mature and immature blood cells



M. Mihailescu *et al*, Appl. Opt. (2011)

Detection of early neuron death



N. Pavillon *et al*, PLOS One (2012)

# Summary



- 3D reconstruction of the amplitude and phase
- Objects are arbitrary
- As fast as your camera
- Simple and inexpensive (about £10K)
  
- Computing is slow (8 hours for 2000 frames at 512 px<sup>3</sup>)
- Cannot get too dense (totally depends on scatterers)
- Fluorescence holography is embryonic (perhaps a reason to move into it...)